EE273 Lecture 2
Wires

September 28, 1998

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Today's Assignment

• Reading
  – Sections 3.3.4 through 3.5.2
  – Complete before class on Wednesday
A Quick Overview

- Wires have
  - resistance
  - capacitance
  - and inductance
- To reason about wires we create models
  - ideal
  - lumped L, R, or C
  - transmission line
- Transmission lines have
  - an impedance $Z_0$
  - a propagation constant, $A$
    - from which we get velocity, $v$
- An LC transmission line is lossless
  - waves travel down the line without loss
  \[ V(x,t) = V(0,t - x/v) \]
- Waves reflect off the ends of a line depending on the termination impedance
  \[ V_R = V_I \left( \frac{Z_0 - R_T}{Z_0 + R_T} \right) \]

Wires in Digital Systems

- Physically wires are
  - Stripguides on printed-circuit cards and backplanes
  - Conductors in cables and cable assemblies
  - Connectors
- We tend to treat them as ideal wires
  - no delay (equipotential)
  - no capacitance, inductance, or resistance
- They are not ideal
- To build reliable systems we need to understand their properties and behavior
Resistance of Wires

- Most real wires have resistance
- Depends on:
  - material (resistivity)
  - length
  - cross section
- Causes:
  - delay
  - loss

\[ R = \frac{\rho L}{A} \]

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho ) (( \text{n}\Omega \cdot \text{m} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>16</td>
</tr>
<tr>
<td>Cu</td>
<td>17</td>
</tr>
<tr>
<td>Au</td>
<td>22</td>
</tr>
<tr>
<td>Al</td>
<td>27</td>
</tr>
</tbody>
</table>

Capacitance of Wires

- Real wires have capacitance
  - line charge
  - parallel plate
  - fringing
- To compute:
  - assume \( Q \)
  - compute \( E \) field
  - integrate to get \( V \)

\[ C = \frac{Q}{V} \]

\[ E = \frac{Q}{2\pi r} \]

\[ C = \frac{2\pi e}{\log \left( \frac{r_o}{r} \right)} \]

\[ C = \frac{2\pi e}{\log \left( \frac{2s}{r} \right)} \]

\[ C = \frac{we}{d} + \frac{2\pi e}{\log \left( \frac{2s}{r} \right)} \]
Inductance of Wires

- Real wires have inductance
  \[ L = \frac{\Lambda}{I} \]
- In a homogenous medium
  \[ CL = \varepsilon\mu \]

Some Example Wires

<table>
<thead>
<tr>
<th>Type</th>
<th>W</th>
<th>R</th>
<th>C</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>On chip</td>
<td>0.6µm</td>
<td>150kΩ/m</td>
<td>200pF/m</td>
<td>600nH/m</td>
</tr>
<tr>
<td>PC Board</td>
<td>150µm</td>
<td>5Ω/m</td>
<td>100pF/m</td>
<td>300nH/m</td>
</tr>
<tr>
<td>24AWG pair</td>
<td>511µm</td>
<td>0.08Ω/m</td>
<td>40pF/m</td>
<td>400nH/m</td>
</tr>
</tbody>
</table>

Scale model of a line has different R, but same L and C per unit length
Wire Models

• In a particular situation, we create a model of a wire that captures the properties we need
  – ideal
  – lumped L, R, or C
  – RC transmission line
  – LC transmission line
  – General LRCG transmission line

• Model to use depends on frequency

\[ f_0 = \frac{R}{2\pi L} \]

LRCG Wire Model

• Model an infinitesimal length of wire, \( dx \), with lumped components
  – L, R, C, and G

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Transmission Line Equations

\[ \frac{\partial V}{\partial x} = RI + L \frac{\partial I}{\partial t} \]

\[ \frac{\partial I}{\partial x} = GV + C \frac{\partial V}{\partial t} \]

\[ \frac{\partial^2 V}{\partial x^2} = RGV + (RC + LG) \frac{\partial V}{\partial t} + LC \frac{\partial^2 V}{\partial t^2} \]

Impedance

- An infinite length of LRCG transmission line has an impedance \( Z_0 \)
- Driving a line terminated into \( Z_0 \) is the same as driving \( Z_0 \)
- In general \( Z_0 \) is complex and frequency dependent
- For LC lines its real and independent of frequency

\[ Z_0 = \left( \frac{R + Ls}{G + Cs} \right)^{\frac{1}{2}} \]

\[ Z_0 = \left( \frac{L}{C} \right)^{\frac{1}{2}} \]

At high frequency (LC lines)
Example, 24AWG Pair

- \( f_0 = 33\text{kHz} \)
- Below \( f_0 \), line is RC
- Above \( f_0 \), line is LC

\[
Z_0 = \left( \frac{0.08 + 400 \times 10^{-9} \times 2\pi f}{40 \times 10^{-9} \times 2\pi f} \right)^{1/2}
\]

Propagation Constant

- Using impedance, we can solve for \( V(s,x) \)
- Propagation is governed by a constant, \( A \)
  - real part is attenuation
  - imaginary part is phase shift
    - velocity

\[
\frac{\partial V(s)}{\partial x} = -(R + Ls)I(s)
\]
\[
= -(R + Ls)V(s)/Z_0
\]
\[
= -[(G + Cs)(R + Ls)]^{1/2}V(s)
\]
\[
V(s, x) = V(s, 0) \exp(-Ax)
\]
\[
A = (G + Cs)(R + Ls)^{1/2}
\]

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Lossless LC Lines

- If \( R \) and \( G \) are negligible
  - line is lossless (no dissipation)
  - governed by the wave equation
- Waves propagate down the line in both directions without distortion
- Line is described by its impedance and velocity
- What happens when the wave gets to the end of the line?

\[
\frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2}
\]

\[
V_f(x,t) = V\left(0, t - \frac{x}{v}\right)
\]

\[
V_r(x,t) = V\left(x_{\max}, t - \frac{x_{\max} - x}{v}\right)
\]

\[
v = (LC)^{\frac{1}{2}}
\]

\[
Z_0 = \left(\frac{L}{C}\right)^{\frac{1}{2}}
\]

Lossless LC Line

Waveform on line is superposition of forward and reverse traveling waves
Driving a Transmission Line

Place waves on the line by driving one end with a source
Assume line is infinite for now

Driving a Line - Equivalent Circuit

Response of line to voltage source depends on previous state of line, \( V_C \)
Termination

- Suppose we drive a unit step, \( U(t) \), on the line
- What happens at the far end?

Termination - Equivalent Circuit
Reflections and The Telegrapher’s Equation

- Incident wave determines \( V_i, I_i \)
- Use equivalent circuit to solve for \( V_T, I_T \)
- Use superposition to calculate \( V_r, I_r \)

\[
\begin{align*}
I_T &= \frac{2V_i}{Z_0 + Z_T} \\
I_r &= I_i - I_T \\
I_r &= \frac{V_i}{Z_0} - \frac{2V_i}{Z_0 + Z_T} \\
I_r &= \frac{V_i}{Z_0} \left( \frac{Z_T - Z_0}{Z_T + Z_0} \right) \\
I_r &= \frac{V_i}{V_i} = \left( \frac{Z_T - Z_0}{Z_T + Z_0} \right)
\end{align*}
\]

Some Common Terminations

- **Open circuit**
- **Matched termination**
- **Short circuit**
Example of Reflections

\[ V_i = 1V \left( \frac{-50}{400+50} \right) = 0.111V \]
\[ k_{SR} = \frac{1000-50}{1000+50} = 0.905 \]
\[ k_{JS} = \frac{400-50}{400+50} = 0.778 \]
Example of Reflections

\[ k_{SR} = \frac{1000 - 50}{1000 + 50} = 0.905 \]

\[ V_I = 1V \left( \frac{50}{400 + 50} \right) = 0.111V \]

\[ k_{rS} = \frac{400 - 50}{400 + 50} = 0.778 \]

<table>
<thead>
<tr>
<th>Vwave</th>
<th>Vline</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vi1</td>
<td>0.111</td>
<td>0</td>
</tr>
<tr>
<td>Vi2</td>
<td>0.101</td>
<td>5</td>
</tr>
<tr>
<td>Vi3</td>
<td>0.078</td>
<td>10</td>
</tr>
<tr>
<td>Vi4</td>
<td>0.071</td>
<td>15</td>
</tr>
<tr>
<td>Vi5</td>
<td>0.055</td>
<td>20</td>
</tr>
<tr>
<td>Vi6</td>
<td>0.050</td>
<td>25</td>
</tr>
<tr>
<td>Vi7</td>
<td>0.039</td>
<td>30</td>
</tr>
<tr>
<td>Vi8</td>
<td>0.035</td>
<td>35</td>
</tr>
<tr>
<td>Vi9</td>
<td>0.027</td>
<td>40</td>
</tr>
</tbody>
</table>

Example of Reflections

![Graph showing reflections](image-url)
Standing Waves

What happens if we drive an open line with a sine wave?
With an arbitrary periodic function (period = round trip)
What if the line is shorted?

Next Time

- Lossy wires
  - attenuation
  - RC transmission lines
- Special transmission lines
  - multi-drop buses
  - balanced lines
  - even and odd-mode propagation