EE273 Lecture 2
Wires

September 28, 1998

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Today's Assignment

- Reading
  - Sections 3.3.4 through 3.5.2
  - Complete before class on Wednesday

A Quick Overview

- Wires have
  - resistance
  - capacitance
  - and inductance
- To reason about wires we create models
  - ideal
  - lumped L, R, or C
  - transmission line
- Transmission lines have
  - an impedance Z_0
  - a propagation constant, A
    - from which we get velocity, v
- An LC transmission line is lossless
  - waves travel down the line without loss

$$V(x,t) = V(0, t - x/v)$$

- Waves reflect off the ends of a line depending on the termination impedance

$$V_R = V_I \left(\frac{Z_0 - R_T}{Z_0 + R_T} \right)$$

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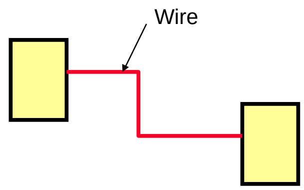
Wires in Digital Systems

- Physically wires are
  - Stripguides on printed-circuit cards and backplanes
  - Conductors in cables and cable assemblies
  - Connectors
- We tend to treat them as ***ideal wires***
  - no delay (equipotential)
  - no capacitance, inductance, or resistance
- They are ***not*** ideal
- To build reliable systems we need to understand their properties and behavior



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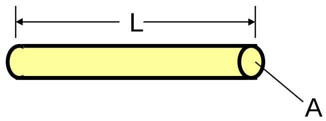
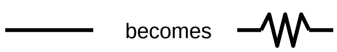
Resistance of Wires

- Most *real* wires have resistance
- Depends on
  - material (resistivity)
  - length
  - cross section
- Causes
  - delay
  - loss



$$R = \frac{\rho L}{A}$$

Material	ρ (nΩ-m)
Ag	16
Cu	17
Au	22
Al	27



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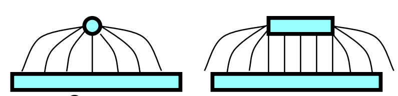
Capacitance of Wires

- Real wires have capacitance
  - line charge
  - parallel plate
  - fringing
- To compute
  - assume Q
  - compute E field
  - integrate to get V

$$C = \frac{Q}{V} \qquad E = \frac{Q}{2\pi r} \qquad C = \frac{2\pi\varepsilon}{\log\left(\frac{r_o}{r_i}\right)}$$

$$C = \frac{2\pi\varepsilon}{\log\left(\frac{s}{r}\right)}$$



$$\frac{2\pi\varepsilon}{\log\left(2s/r\right)} \qquad C = \frac{w\varepsilon}{d} + \frac{2\pi\varepsilon}{\log\left(2s/r\right)}$$

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Inductance of Wires

- Real wires have inductance

$$L = \frac{\Lambda}{I}$$

- In a homogenous medium

$$CL = \varepsilon\mu$$

Some Example Wires

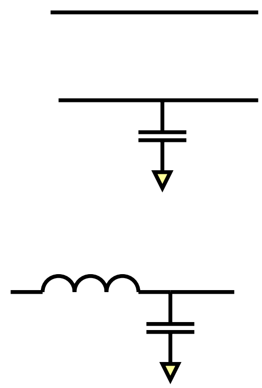
Type	W	R	C	L
On chip	0.6µm	150kΩ/m	200pf/m	600nH/m
PC Board	150µm	5Ω/m	100pf/m	300nH/m
24AWG pair	511µm	0.08Ω/m	40pf/m	400nH/m

Scale model of a line has different R, but same L and C per unit length

Wire Models

- In a particular situation, we create a *model* of a wire that captures the properties we need
  - ideal
  - lumped L, R, or C
  - RC transmission line
  - LC transmission line
  - General LRCG transmission line
- Model to use depends on frequency

$$f_0 = \frac{R}{2\pi L}$$



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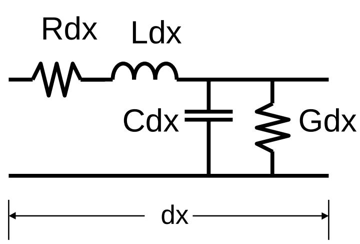
LRCG Wire Model

- Model an *infinitesimal* length of wire, dx, with lumped components
  - L, R, C, and G



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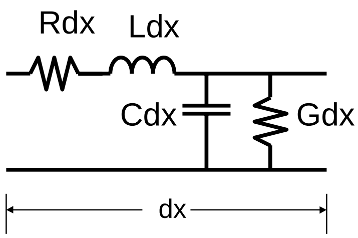
Transmission Line Equations



Drop across R and L $\frac{\partial V}{\partial x} = RI + L\frac{\partial I}{\partial t}$

Current into C and G $\frac{\partial I}{\partial x} = GV + C\frac{\partial V}{\partial t}$

$$\frac{\partial^2 V}{\partial x^2} = RGV + (RC + LG)\frac{\partial V}{\partial t} + LC\frac{\partial^2 V}{\partial t^2}$$

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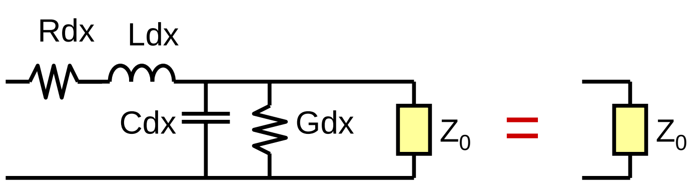
Impedance

- An infinite length of LRCG transmission line has an *impedance* Z_0
- Driving a line *terminated* into Z_0 is the same as driving Z_0
- In general Z_0 is complex and frequency dependent
- For LC lines its real and independent of frequency



$$Z_0 = \left(\frac{R + Ls}{G + Cs}\right)^{\frac{1}{2}}$$

$$Z_0 = \left(\frac{L}{C}\right)^{\frac{1}{2}}$$

At high frequency (LC lines)

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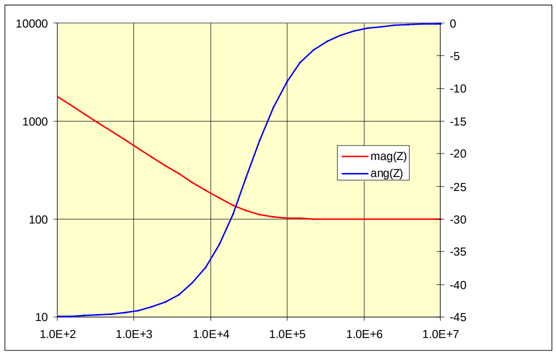
Example, 24AWG Pair

- f_0 = 33kHz
- Below f_0, line is RC
- Above f_0, line is LC

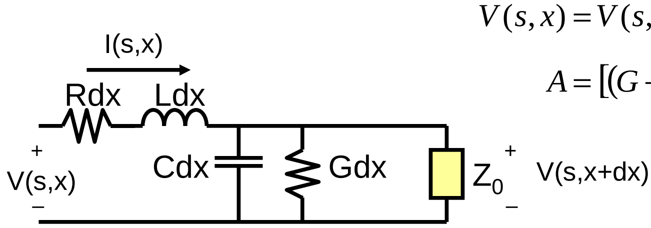
$$Z_0 = \left(\frac{.08 + 400\times10^{-9}\times2\pi fj}{40\times10^{-9}\times2\pi fj} \right)^{\frac{1}{2}}$$



Propagation Constant

- Using impedance, we can solve for V(s,x)
- Propagation is governed by a constant, A
  - real part is attenuation
  - imaginary part is phase shift
    - velocity^{-1}

$$\frac{\partial V(s)}{\partial x} = -(R+Ls)I(s)$$
$$= -(R+Ls)V(s)/Z_0$$
$$= -[(G+Cs)(R+Ls)]^{\frac{1}{2}}V(s)$$
$$V(s,x) = V(s,0)\exp(-Ax)$$
$$A = [(G+Cs)(R+Ls)]^{\frac{1}{2}}$$



Lossless LC Lines

- If R and G are negligible
  - line is lossless (no dissipation)
  - governed by the *wave equation*
- Waves propagate down the line in both directions without distortion
- Line is described by its impedance and velocity
- What happens when the wave gets to the end of the line?

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

$$V_f(x,t) = V\left(0, t - \frac{x}{v}\right)$$

$$V_r(x,t) = V\left(x_{\max}, t - \frac{x_{\max} - x}{v}\right)$$

$$v = (LC)^{-\frac{1}{2}}$$

$$Z_0 = \left(\frac{L}{C}\right)^{\frac{1}{2}}$$

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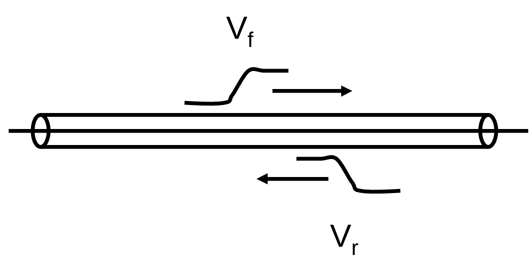
Lossless LC Line



Waveform on line is superposition of forward and reverse traveling waves

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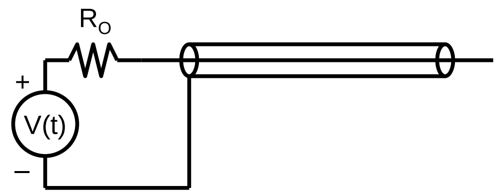
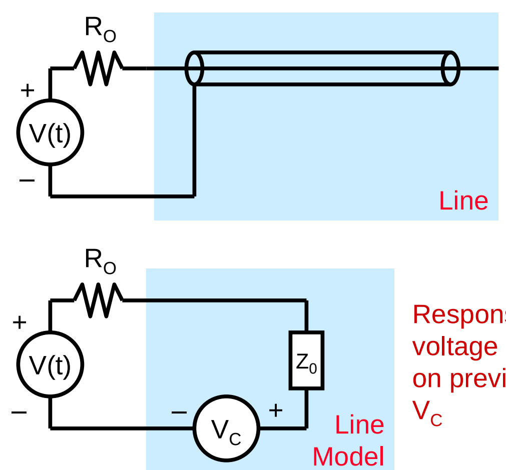
Driving a Transmission Line



Place waves on the line by driving one end with a source
Assume line is infinite for now

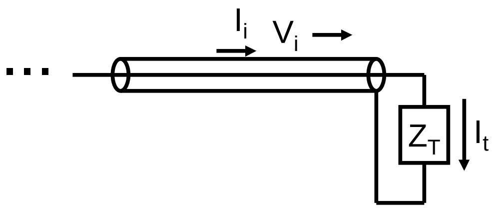
Driving a Line - Equivalent Circuit



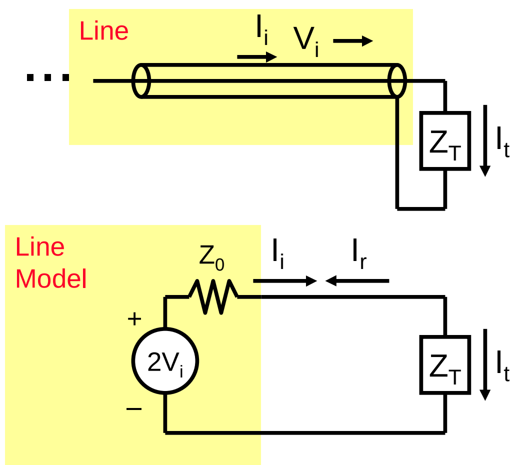
Response of line to voltage source depends on previous state of line, V_C

Termination

- Suppose we drive a unit step, U(t), on the line
- What happens at the far end?



Termination - Equivalent Circuit



Reflections and The Telegrapher's Equation

- Incident wave determines V_i, I_i
- Use equivalent circuit to solve for V_T, I_T
- Use superposition to calculate V_r, I_r

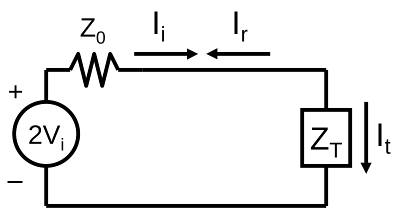
$$I_T = \frac{2V_i}{Z_0 + Z_T}$$

$$I_r = I_i - I_T$$

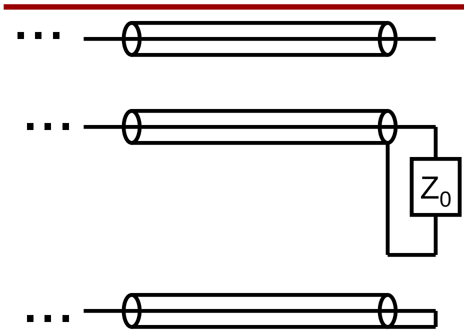
$$I_r = \frac{V_i}{Z_0} - \frac{2V_i}{Z_0 + Z_T}$$

$$I_r = \frac{V_i}{Z_0}\left(\frac{Z_T - Z_0}{Z_T + Z_0}\right)$$

$$\frac{I_r}{I_i} = \frac{V_r}{V_i} = \left(\frac{Z_T - Z_0}{Z_T + Z_0}\right)$$



Some Common Terminations

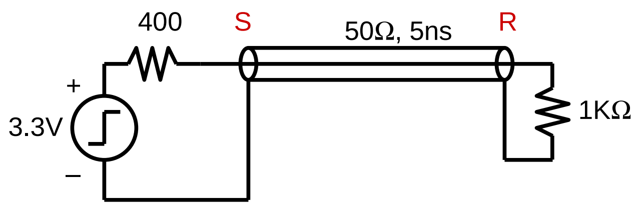


Open circuit

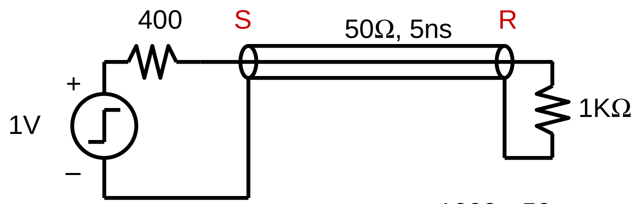
Matched termination

Short circuit

Example of Reflections



Example of Reflections

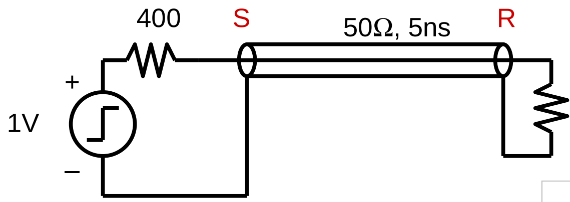


$$V_i = 1V\left(\frac{50}{400+50}\right) = 0.111V$$

$$k_{rR} = \frac{1000-50}{1000+50} = 0.905$$

$$k_{rS} = \frac{400-50}{400+50} = 0.778$$

Example of Reflections



$$k_{rR} = \frac{1000-50}{1000+50} = 0.905$$

$$V_i = 1V\left(\frac{50}{400+50}\right) = 0.111V$$

$$k_{rS} = \frac{400-50}{400+50} = 0.778$$

	Vwave	Vline	t
Vi1	0.111	0.111	0
Vr1	0.101	0.212	5
Vi2	0.078	0.290	10
Vr2	0.071	0.361	15
Vi3	0.055	0.416	20
Vr3	0.050	0.465	25
Vi4	0.039	0.504	30
Vr4	0.035	0.539	35
Vi5	0.027	0.566	40

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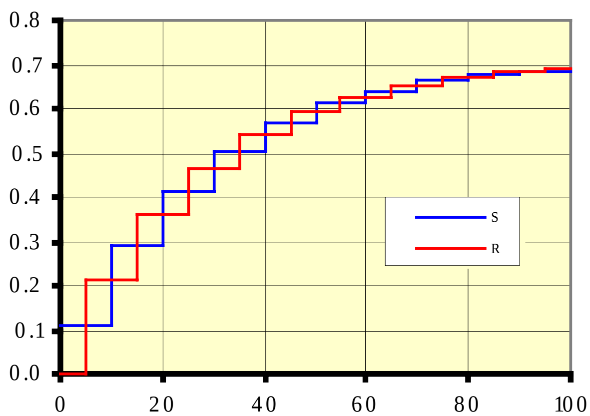
Example of Reflections



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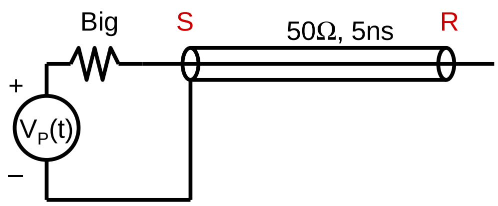
Standing Waves



What happens if we drive an open line with a sine wave?

With an arbitrary periodic function (period = round trip)

What if the line is shorted?

Next Time

- Lossy wires
  - attenuation
  - RC transmission lines
- Special transmission lines
  - multi-drop buses
  - balanced lines
  - even and odd-mode propagation