

Problem Set #6 Solution

SPICE deck files are in the directory /usr/class/ee273/spice/hw6 directory.

Problem 9-6: Sequential Phase-Only Comparator

The following table shows the transition table of the comparator logic:

$$\begin{aligned}a0 &= a * x_b * b0_b \\b0 &= b * x_b * a0_b \\x &= (a * b) + ((a + b) * x)\end{aligned}$$

a0	b0	x	ab			
			00	01	10	11
0	0	0	000	010	100	111
0	0	1	000	001	001	001
0	1	0	000	010	000	011
0	1	1	000	001	001	001
1	0	0	000	000	100	101
1	0	1	000	001	001	001
1	1	0	000	000	000	001
1	1	1	000	001	001	001

The each row indicated by (a0, b0, x) indicates the current state, each column of (a,b) indicate the current input, and each entry of the table indicates the next state (a0, b0, x). Since this is an asynchronous system, there're no latches between the current state (a0,b0, x), and the next state (a0, b0, x). Therefore, one change in input may cause multiple state transitions. For example, if the current state is (1,0,0) and the input changes from (1,0) to (1,1), state first changes to (1,0,1), but the entry at row (1,0,1) and column (1,1) suggests that the state will now transition to 001, at which state it will remain until input changes again.

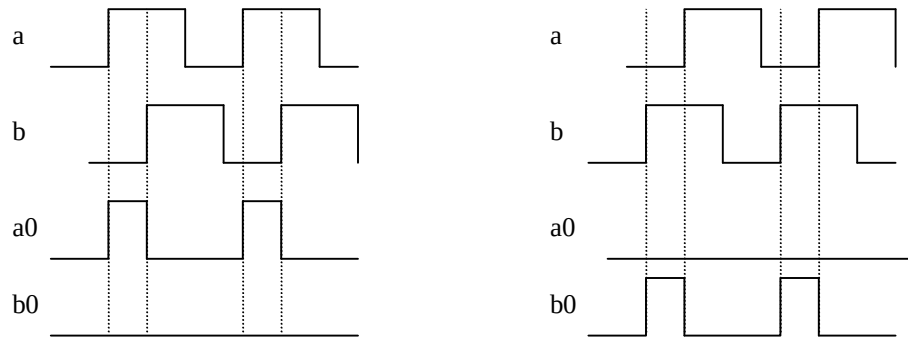
Therefore, now there are stable states and unstable ones, and bold entries indicate the stable ones.

Basically, if the next state is equal to the current state, the state will not transition any more for the given input, and it is stable. If inputs are constrained so that only one bit changes at a time, the construction and analysis of such an asynchronous system is pretty straightforward. But in this case, two inputs bits can change simultaneously. (In fact, the closed loop system in which this comparator will be used forces them to change simultaneously.) Therefore, the actual behavior is a bit more tricky since it depends on delay of each input transition to output transition etc. As we'll see later, even if the two input bits don't change simultaneously, if the transitions happen in a short amount of time compared to the delay of the logic, it looks as if the bits changed simultaneously. This can be a good thing or bad thing, as will be shown later.

Just by following the transition table above, we can see the general behavior of this comparator. Basically, when input bits are (0,0), the comparator goes into reset state of (0,0,0). It stays there until one of the input bits goes high, at which point one of the a0 and b0 lines is raised. This continues until the other edge goes up. Then both a0 and b0 lines go low, and x goes high, indicating that the comparator has registered both edges. This state is (0,0,1), which may be labeled 'wait' state, since it sits there until both the input bits become zero, at which time the cycle repeats. Careful observation also reveals that this comparator depends on the facts that there's at least some overlap between high a and high b regions and also between low a and low b regions. If the a and b waveforms do not have overlap 1-regions, (which would be the case if the duty cycle is less than 50% and the two input signals have large phase offset,) the table predicts that a0 and b0 will have equal duty cycles. Since the actual phase offset information is gotten by subtracting the b0 duty cycle from a0 duty cycle, this non-overlapping a and b waveforms result in phase reading of zero, which is an error. Also, when a and b have no overlapping low regions, (which would be the case if duty cycle > 50% and phase offset so that (a,b) is not equal to (0,0) at any time,) we can see that the system will get stuck in the 'wait' state, since the comparator is reset by (a,b) being (0,0). Other than these two

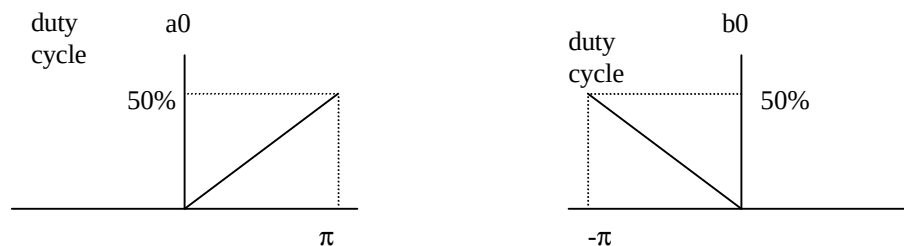
requirements, the frequencies of the two inputs should be matched as well. The feedback loop will probably correct any small frequency error, but the phase reading will be meaningless unless the two inputs are at similar frequencies.

Two example waveforms predicted by the table are shown below.



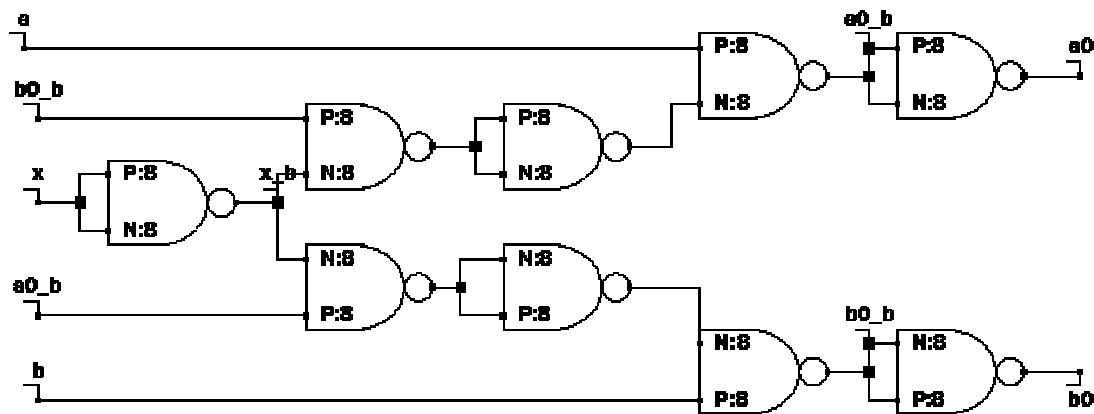
As shown, a0 will indicate that a0 is leading (starting from the reset state), and b0 will indicate that b0 is leading. Also, the duty cycle of these two signals show the amount of phase offset. When they're exactly matched, meaning a and b go high and low at the same time, a0 and b0 will both stay at 0, if we assume the logic delay is none and transitions are instantaneous. On the table, this corresponds to state transitions of (0,0,0) $\rightarrow$ (1,1,1) $\rightarrow$ (0,0,1) $\rightarrow$ (0,0,0), which confirms that a0 and b0 are zero at all stages. But again, the logic delay will show slightly different behavior, as will be shown on the HSPICE simulation.

From the example waveforms shown, we can see the dynamic range of this comparator. Assuming one period of the periodic waveform corresponds to 2π , and positive phase means a is leading, the response looks as follows:

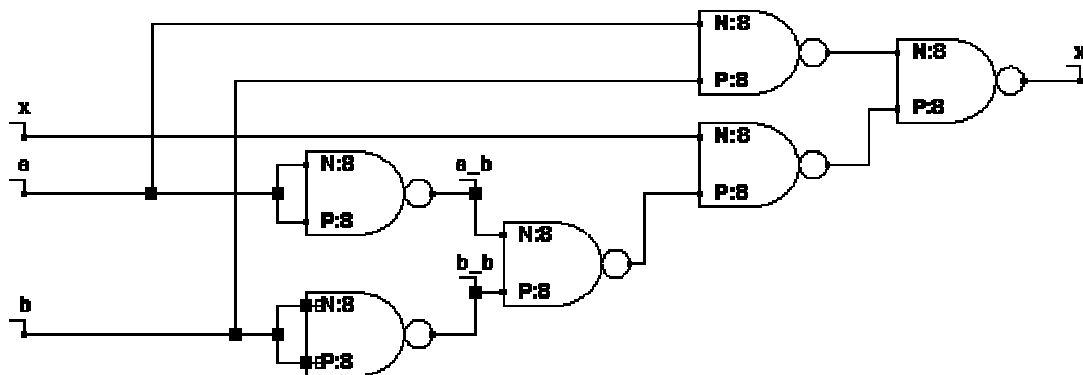


Therefore, $a0 - b0$ gives a linear phase response of this comparator. By the way, when the phase offset is exactly $\pm\pi$, meaning they're complement of each other, funny thing happens. Over a broader region around that area, it can be deduced that the phase response will wrap around, so that a0's duty cycle of 50% will be small offset away from b0's duty cycle of 50%. However, a metastable point exists when they're exactly π apart. In the table, it means that the comparator will stay in 'wait' state forever, since the inputs are never (0,0) and they switch from (1,0) and (0,1) only. 'Wait' state is a stable state for both of these inputs, and the comparator never gets out of it. This has nothing to do with the logic delay, since it strictly depends on the input transition. This is a bad thing unless phase lock of $\pm\pi$ radians is also okay, but hopefully, this exact phase offset will not occur when the closed loop control is doing its work well. Notice that this state is called meta-stable, since any deviation from it will force the system toward phase offset of zero, which is the only truly stable point of the system.

Now, let's consider how to construct this phase comparator using only two input NAND gates. One possible solution is shown below, which is gotten by pushing bubbles around. Notice that inverters were obtained by tying both inputs of NAND gates together. I didn't really worry about gate sizing at all, and simply used the default gates sizes.

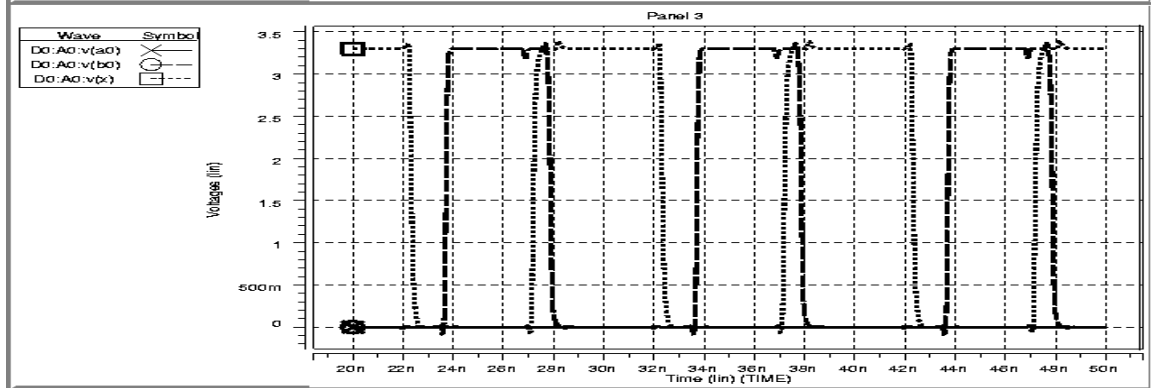
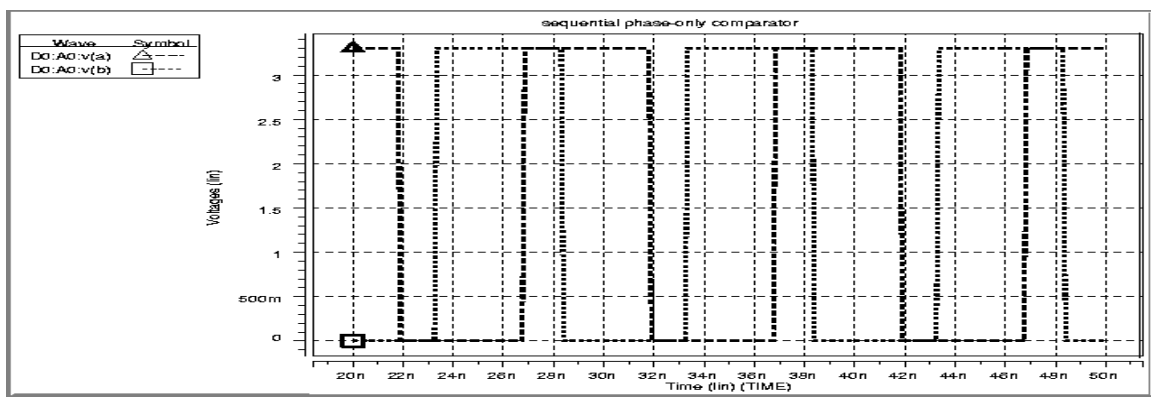
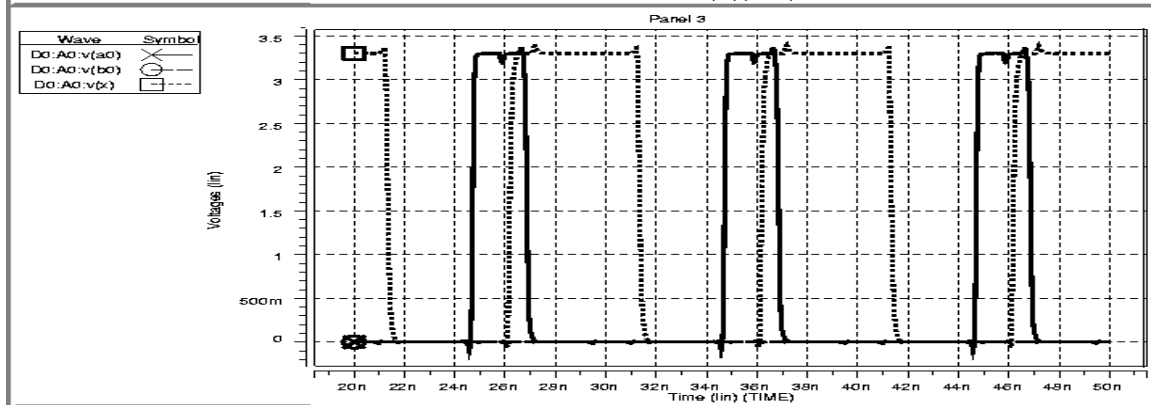
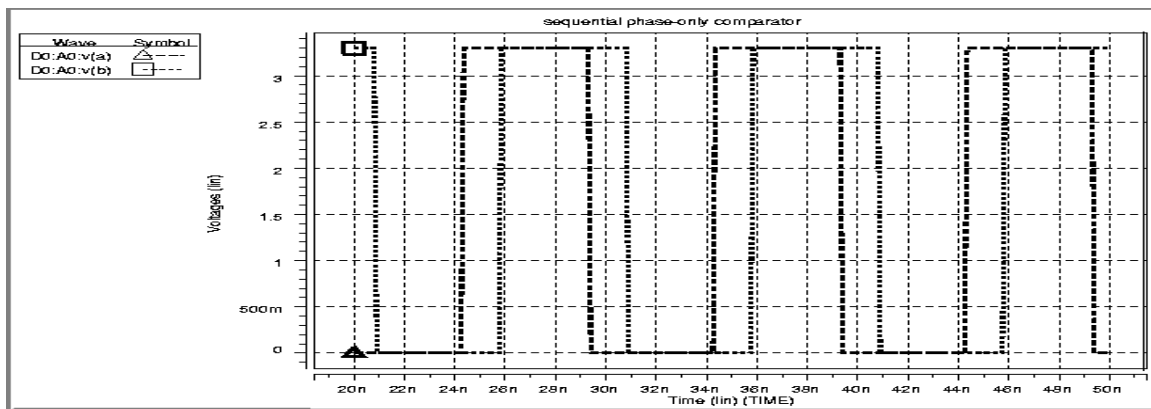


$$\begin{aligned} a0 &= a \text{ AND } x_b \text{ AND } b0_b \\ b0 &= b \text{ AND } x_b \text{ AND } a0_b \\ x &= (a \text{ AND } b) \text{ OR } ((a \text{ OR } b) \text{ AND } x) \end{aligned}$$

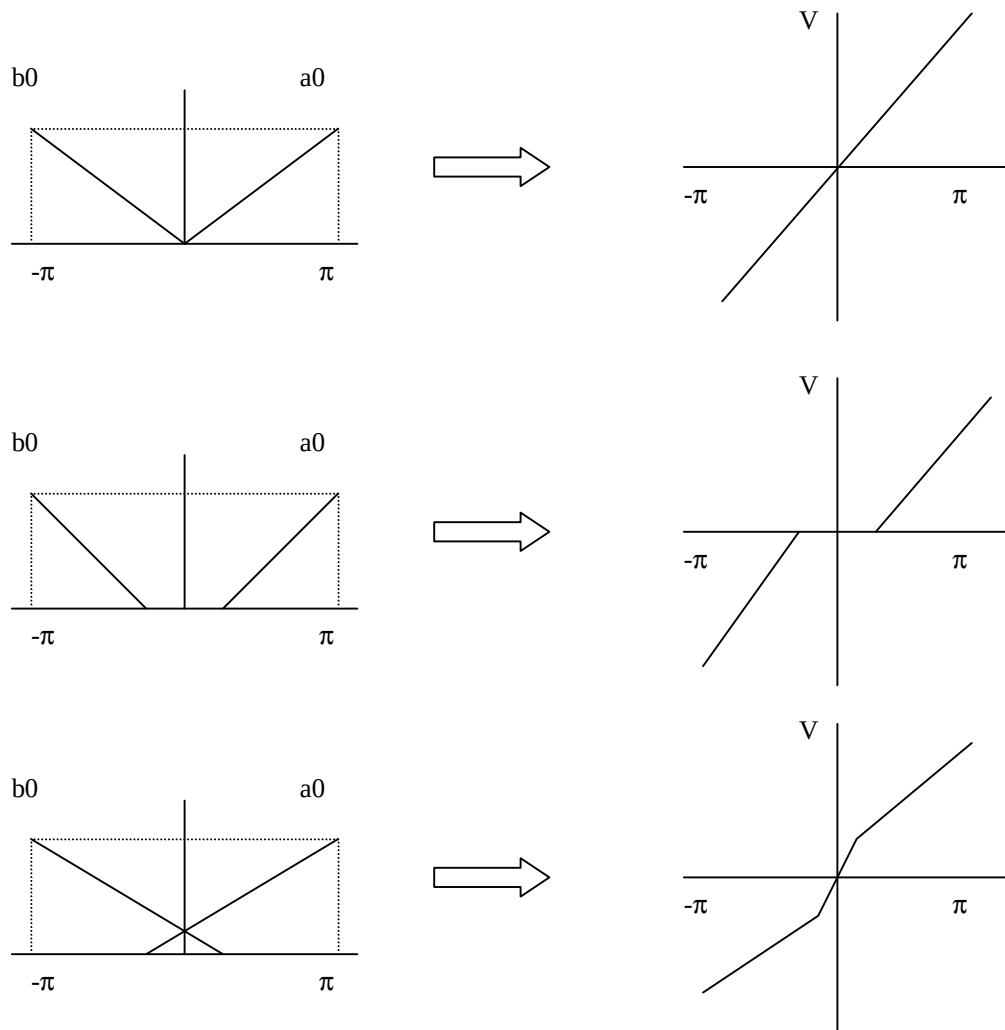


The following page shows the HSPICE simulation results. The upper plot shows the waveforms when a arrives earlier than b by about 0.15π , and the lower plot shows the waveforms when b arrives earlier than a by about 0.35π . For both of these plots, the top graph is the input waveforms, and the bottom one is the output (state) waveforms. As shown, the period of the pulse waveforms is 10nsec, corresponding to 100Mhz frequency.

Even though the edges of the output waveforms are not very sharp due to non-optimal gate sizings, and there're some noticeable delays, the waveforms closely resembles the expected waveforms. That is, when a is early, $a0$ goes high for a while, and when b is early, $b0$ goes high for a while. Also, the duty cycle is approximately proportional to the offset amount.



Even for relatively ideal response as shown on the previous page, the effect of logic delay is evident. For example, when a is leading, a_0 doesn't go high at the same moment a goes high, and a_0 doesn't go low at the same moment b goes high. Also, the overlap between x and a_0 is quite visible, which is not predicted by naïve interpretation of the state transition table. Now, my concern is whether this comparator has a dead zone or not. Dead zone is where there is non-zero offset which the comparator can't see. We've already seen one kind of dead zone around phase offset of $\pm\pi$. But since this is a meta-stable state, we can count on the fact that small disturbance will eventually bring the system out of this state. Besides, assuming the closed loop does its job well, we will not spend much time around this region at all. The more crucial region is around phase of zero. Since we want zero offset, the comparator response around this region must be very sensitive. Now, three possibilities of a_0 and b_0 response and its effect on the comparator response is shown below. Note that comparator response is simply a_0 response minus b_0 response.



The first case shows the ideal case, when we neglect any delay. This gives a perfectly linear comparator response (which is not necessarily good, as explained in the lecture.) However, due to process variations, it's not a good idea to rely on this ideal behavior. For example, something like the second case can happen, where there's some area when neither a_0 and b_0 are high for small offset. For this case, the comparator will say that there's no phase error even when there is, which prevents accurate phase locking from happening. So, second case is definitely a bad thing. What we want is something like the third case, which gives some margin to tolerate adverse process variation toward direction of the second case. For this, we