EE273 Digital Systems Engineering
Midterm Exam

February 12th, 2001

(Total time = 120 minutes, Total Points = 100)

Name: (please print)_____SOLUTION________________

In recognition of and in the spirit of the Stanford University Honor Code, I certify that I will neither give nor receive unpermitted aid on this exam.

Signature:_______________________________________

This examination is open notes open book. You may not, however collaborate in any manner on this exam. You have two hours to complete the exam. Please do all of your work on the exam itself. Attach any additional pages as necessary.

Before starting, please check to make sure that you have all 9 pages. (11 for the solution)

1		**40**
2		**20**
3		**25**
4		**15**
Total		**100**

Problem 1: Short Answer (40 Points: 10 questions, 4 points each)

A. Consider a stripguide transmission line with an impedance of 50Ω, a capacitance of 100pF/m, and a delay of 4ns. Suppose you insert a 10pF capacitor across the line (between the line and GND) every 1cm. What is the impedance of the line with the capacitors inserted?

$$Z = \sqrt{\frac{L}{C}} = \sqrt{\frac{L}{C_0 + C_1}} = \sqrt{\frac{L}{C_0}}\sqrt{\frac{C_0}{C_0 + C_1}} = (50\Omega)\sqrt{\frac{100}{100 + 1000}} = (50\Omega)(0.302) = 15.1\Omega$$

B. For the transmission line of (A), what is the delay of the line with the capacitors inserted?

$$t_d = d\sqrt{LC} = d\sqrt{L(C_0 + C_1)} = d\sqrt{LC_0}\sqrt{\frac{C_0 + C_1}{C_0}}$$

$$= (4ns)\sqrt{\frac{100 + 1000}{100}} = (4ns)(3.32) = 13.2ns$$

C. For the transmission line of (A), including the additional capacitors, what is the fastest rise time that can be applied such that modeling the line with capacitors as a uniform line gives an accurate result?

$$L = Z^2 C = (50\Omega)^2 \left(100 \tfrac{\text{pf}}{\text{m}}\right) = 250 \tfrac{\text{nH}}{\text{m}}$$

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(250 \tfrac{\text{nH}}{\text{m}}\right)\left(1.1 \tfrac{\text{nF}}{\text{m}}\right)}} = 6.03 \times 10^7 \tfrac{\text{m}}{\text{s}}$$

$$t_s = \frac{d}{v} = \frac{0.01\text{m}}{6.03 \times 10^7 \tfrac{\text{m}}{\text{s}}} = 166\text{ps}$$

$$t_r \geq 4t_s = (4)(166\text{ps}) = 663\text{ps}$$

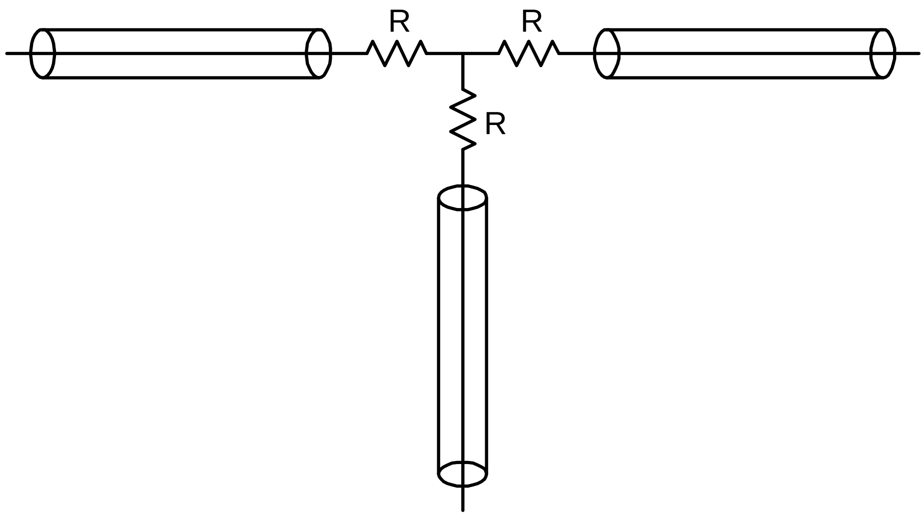
D. Consider a differential transmission line. Which is higher: the even-mode impedance or the odd-mode impedance?

Even mode – since $\sqrt{\frac{L + M}{C - C_M}} \geq \sqrt{\frac{L - M}{C + C_M}}$

E. If the space between the two signal conductors on a differential transmission line is reduced, what happens to the odd-mode impedance? (a) it increases, (b) it decreases, (c) it stays the same.

(b) decreases since C_M *and M increase*

F. Sketch a resistor network that allows three 50Ω transmission lines to be connected together so that a wave arriving at the network from any one of the lines causes two equal waves to be propagated down the other two lines with no reflection back into the first line. Make sure to include all component values.



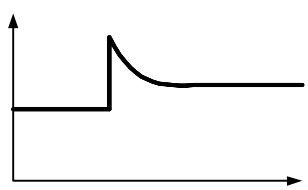
$$R + (R + Z_0) \| (R + Z_0) = Z_0$$

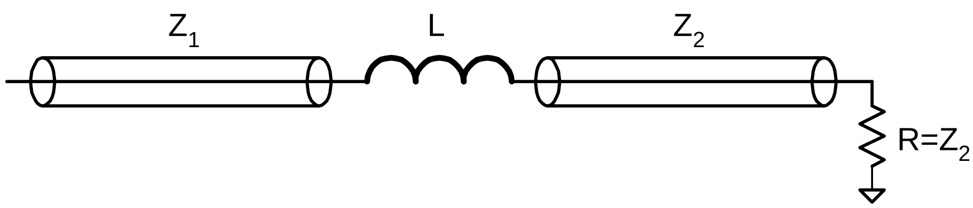
$$R + 0.5(R + Z_0) = Z_0$$

$$1.5R = 0.5Z_0$$

$$R = \frac{0.5}{1.5} Z_0 = \frac{1}{3} Z_0 = 16.7\Omega$$

G. Sketch a qualitative circuit that will produce the following trace on a time-domain reflectometer (TDR). Just draw the type of components. Do not include numerical component values.





H. A pair of 45Ω lines have a near end (reverse) crosstalk coefficient k_{rx} = 0.1 and a far-end (forward) crosstalk coefficient of k_{fx} = 0. Both lines are terminated at both ends with 55Ω resistors. As a fraction of signal swing, what is the noise due to crosstalk on each line?

*Near end crosstalk gives a wave at the near end of the line with a magnitude of 0.1DV. This reflects off the terminator with k_r = 10/100 = 0.1 giving an overall contribution of 0.01. Also, the incident wave reflects off the far end with k_r = 0.1 which directly induces near end crosstalk at the far end with a magnitude of 0.01. Thus the total crosstalk coefficient is **k_{NX} = 0.02**.*

I. A system has a signal swing of 100mV and bounded noise sources (fixed and proportional) that total 30mV. If there is 10mV RMS of Gaussian noise, what will the BER of the system be? (a simplified expression is fine, you need not give a numerical answer).

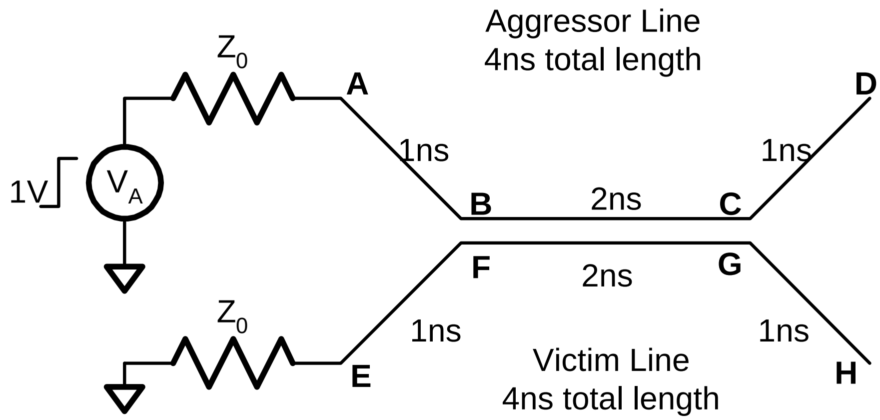
the gross margin, V_{GM} = 50mV. Subtracting the 30mV of bounded noise gives a net margin, V_{NM} = 20mV. With V_G = 10mV of Gaussian noise, we have VSNR = V_{NM} /V_G = 2 and BER = exp(-VSNR^2/2) = 0.135.

J. Making the source impedance of a transmission system infinite reduces what type(s) of noise?

Signal return crosstalk at the transmitter and transmitter power supply noise.

Problem 2: Transmission Lines (20 Points)

Consider the pair of coupled transmission lines shown below. The coupled section of the pair (segments BC and FG) has a near-end crosstalk coefficient k_{rx} of 0.1 and a far-end crosstalk coefficient, k_{fx} of 0. The aggressor line is driven directly by a 1V step source with a rise time of 100ps and a matched source impedance. The far end of both lines (points D and H) are left open. The victim line is terminated with a matched impedance at the near end.



Using this information, sketch and dimension the voltage waveform at the far-end of the victim line (point H). You may ignore any effects that lead to waves with less than 10mV amplitude.

The response is a 100mV 4ns pulse from 6ns to 10ns.

The source injects a 500mV forward traveling wave at point A. When this wave reaches B (at 1ns) it induces a 50mV reverse traveling wave at F with a pulse width of 4ns (the round trip from B to C and back). This 4ns pulse is absorbed at the termination of E.

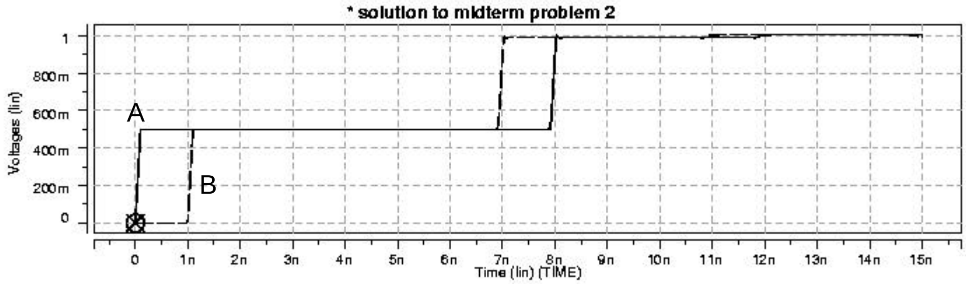
The forward wave reaches D at 4ns and completely reflects generating a 500mV reverse traveling wave. At 5ns this wave reaches C where it induces a 50mV forward traveling wave at G with a pulse width of 4ns (the round trip from C to B and back). At 6ns this reverse traveling wave reaches H and completely reflects. This gives a 100mV pulse from 6ns to 10ns on H.

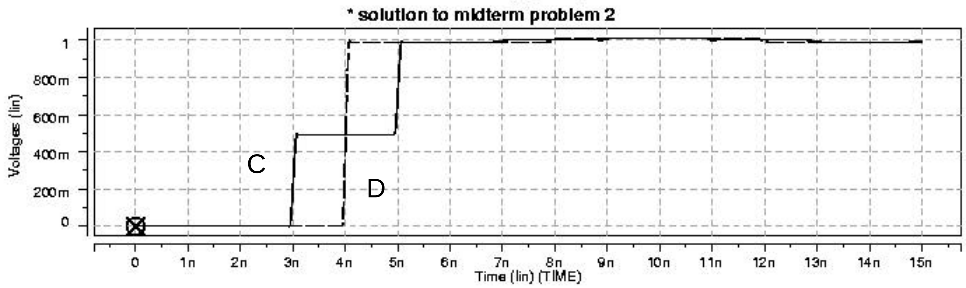
The 50mV reverse traveling wave reflected from H returns over the victim line and is absorbed into the terminator at E.

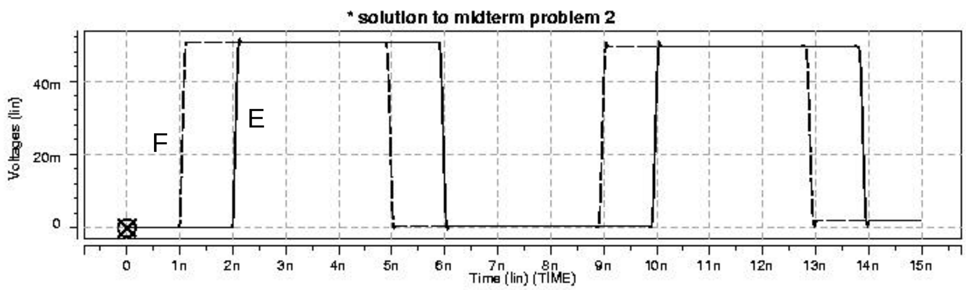
Similarly, the 500mV reverse traveling wave that passed C at 5ns reaches B at 7ns. At this point the injection of reverse crosstalk stops, but it takes until 9ns for the crosstalk injected to reach G and 10ns for the last of the crosstalk to reach H – hence the end of the pulse at 10ns. The 500mV reverse traveling wave is absorbed into the terminator at A at 8ns.

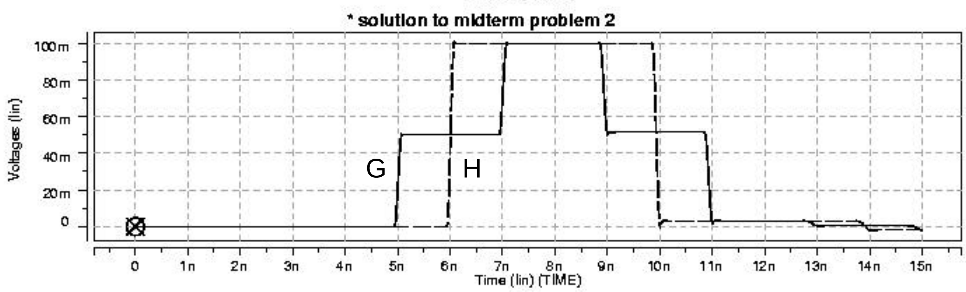
There is secondary crosstalk induced at C by the 50mV reverse traveling wave at G, but its magnitude is 5mV, below our threshold of concern.

A SPICE plot showing the voltages at all points is attached.


* solution to midterm problem 2
Voltages (lin)
1
800m
600m
400m
200m
0
A
B
0 1n 2n 3n 4n 5n 6n 7n 8n 9n 10n 11n 12n 13n 14n 15n
Time (lin) (TIME)

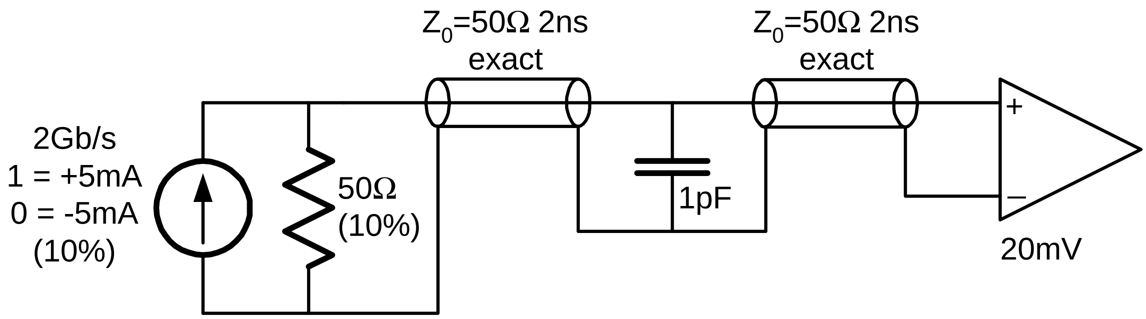

* solution to midterm problem 2
Voltages (lin)
1
800m
600m
400m
200m
0
C
D
0 1n 2n 3n 4n 5n 6n 7n 8n 9n 10n 11n 12n 13n 14n 15n
Time (lin) (TIME)


* solution to midterm problem 2
Voltages (lin)
40m
20m
0
F
E
0 1n 2n 3n 4n 5n 6n 7n 8n 9n 10n 11n 12n 13n 14n 15n
Time (lin) (TIME)


* solution to midterm problem 2
Voltages (lin)
100m
80m
60m
40m
20m
0
G
H
0 1n 2n 3n 4n 5n 6n 7n 8n 9n 10n 11n 12n 13n 14n 15n
Time (lin) (TIME)

Problem 3: Signaling and Noise Analysis (25 points total)

Consider the 2Gb/s (t_{bit} = 500ps) bipolar current-mode signaling system shown below. At nominal levels, a logic "1" is represented with 5mA of current drive and a logic 0 is represented with -5mA of drive. The actual transmitter levels are within 10% of these nominal levels. The transmitter has a rise/fall time of 250ps. The line is terminated at the source only with a matched impedance with 10% tolerance. Midway down the 4ns 50Ω line a connector introduces a large 1pF lumped capacitance. The receiver has a combined sensitivity and offset voltage of 20mV. In addition, there is a 10mV Gaussian noise source (not shown) adding noise to the line.



A. (10 points) List all of the bounded, proportional noise sources that affect this system and give the magnitude of each as a fraction of signal swing. (Hint: you may ignore all forms of crosstalk and you may ignore all reflections after the second. Make sure to consider all of the effects of the 1pF capacitance.).

1. Transmitter offset, k=0.05, in the worst case a 1 or a 0 is signaled with 4.5mA rather than 5, a 0.5mA difference out of a 10mA swing, so k = 1mA/10mA = 0.05.
2. Reduced swing due to mismatched transmitter termination, k=0.026, 5mA should generate 125mV, with a 45W terminator, it generates only 118mV. k = (125-118)/250 = 0.026. Note that it is impossible to get a combined 0.076 error from transmitter offset and mismatched source termination. The worst case is a 4.5mA current into a 45W termination generating 107mV, in this case the combined k for transmitter offset and source termination is k = (125-107)/250 = 0.074 rather than 0.076 (full credit is given for this solution as well). Note that even though the full swing is 500mV, we compute the proportional noise for the 250mV incident wave. The reflection from the open end of the line doubles both the signal and the noise, so the proportion, k, remains the same.
3. ISI due to reflection from the end of the line reflecting off the capacitor. The capacitor has a time constant of t = (1pF)(25W) = 25ps giving a reflection of k_{rc} = t/t_r = 25ps/250ps = 0.1.
4. ISI due to reflections from the end of the line reflecting off the mismatched source termination, k_{rr} = 0.05.
5. ISI due to reflections from the capacitor reflecting off the mismatched source termination. Here k= $k_{rc}k_{rr}$ = (0.1)(0.05) = 0.005, which is small enough to ignore.

So the total proportional noise is 0.074+0.1+0.05+0.005 = 0.23

B. (5 points) List all of the bounded, fixed noise sources that affect this system and give the magnitude for each in millivolts.

Receiver offset, 20mV

C. (5 points) Compute the net margin, VSNR, and BER for this signaling system.

Gross margin is 250mV
*Proportional noise is 0.23*500mV = 114mV*
Fixed noise 20mV
Net margin is 250-114-20 = 116mV

The Gaussian noise is doubled at the open end of the line, so the total Gaussian noise is 20mV RMS

Thus VSNR = 116/20 = 5.78

BER $<= \exp(-VSNR^2/2) = 5.5$ ´ 10^{-8}

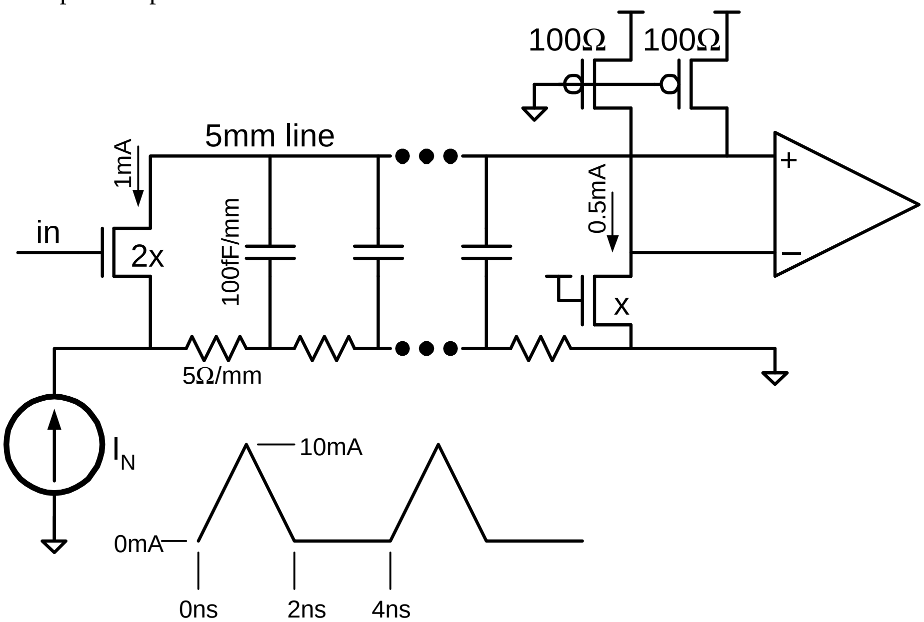
D. (5 points) What if the same system were used to send 2-bits per baud at 1Gbaud using the encoding 0 = -5mA, 1 = -1.33mA, 2 = +1.33mA, and 3 = +5mA (all with 10% tolerance). Recalculate the noise sources from (A) and (B), if any, that change as a result of this change, and recalculate the net margin, VSNR, and BER.

The noise sources do not change, so we still have $k_N = 0.23$. *This is too large for a 4-level system where the gross margin is only 0.167 of signal swing.*

Thus, there is no net margin and the BER is 0.5.

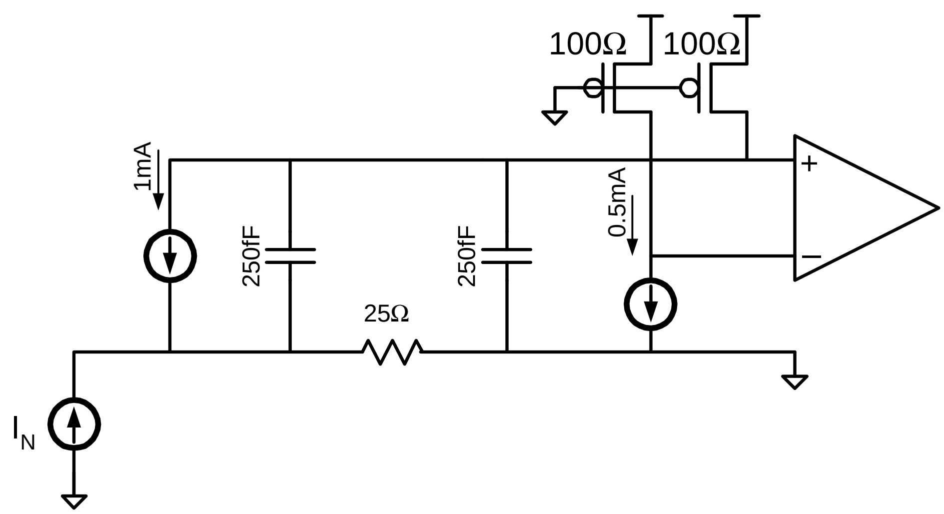
Problem 4: Signaling over Lumped Loads (15 Points Total)

Consider the system shown below for signaling over a capacitive on-chip line. A 1mA current driver signals over a 5mm line with a 1 with 1mA current and a 0 with no current. The receiver converts this current to a voltage using a 100-Ohm PFET resistor. A reference is generated at the receiver using an 0.5mA current source and a second 100-Ohm PFET resistor. Over the 5mm from the transmitter to the receiver the ground supply has a resistance of 5Ω/mm (25Ω total). There is a distributed capacitance from the signal line to the ground supply of 100fF/mm (500fF total). The ground supply carries a noise current I_N with a peak amplitude of 10mA and the waveform shown.

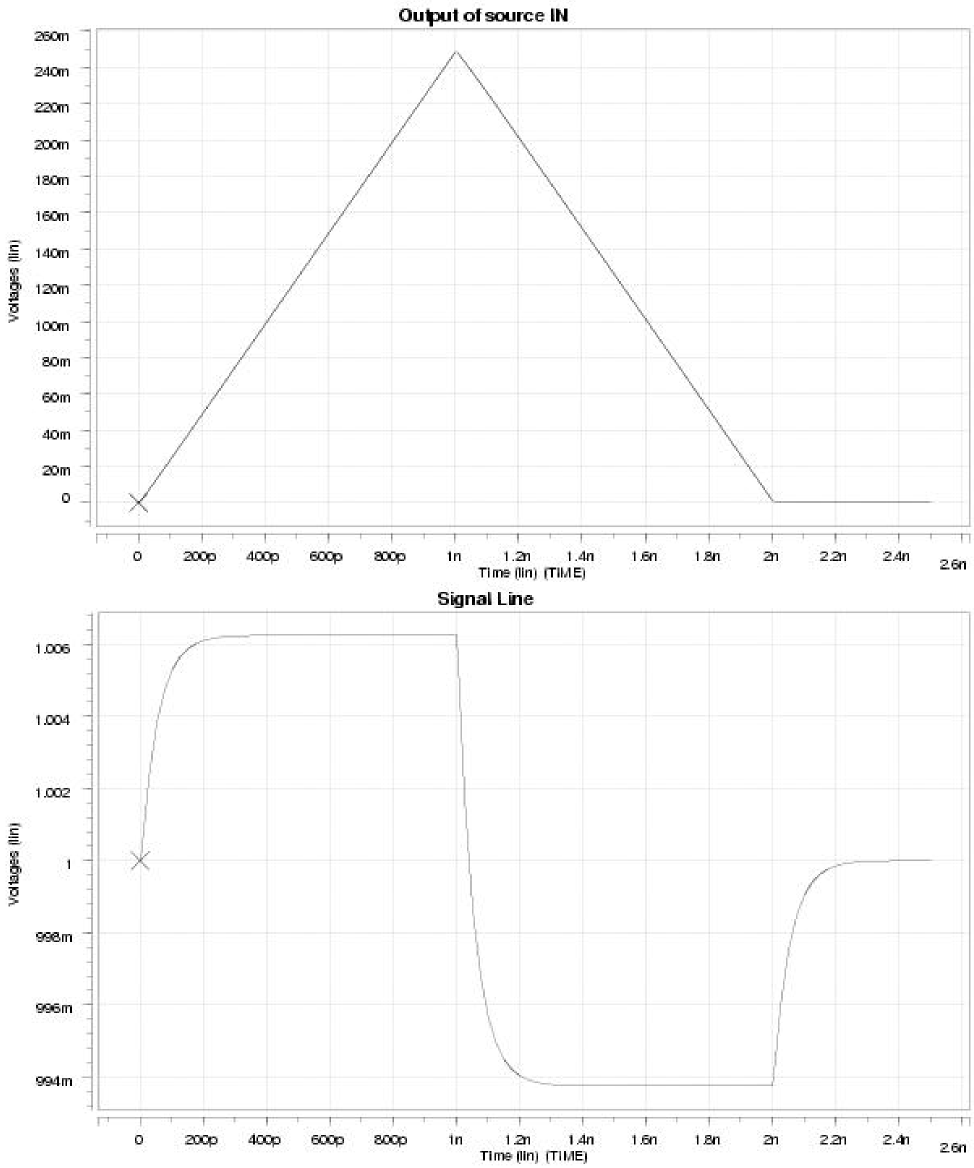


A. (9 points) What is the magnitude (in mV) of the noise at the receiver due to the noise current I_N? (Hint: approximate the distributed RC line with a Pi or Tee network).

Replacing the distributed RC with a PI model we get the following circuit

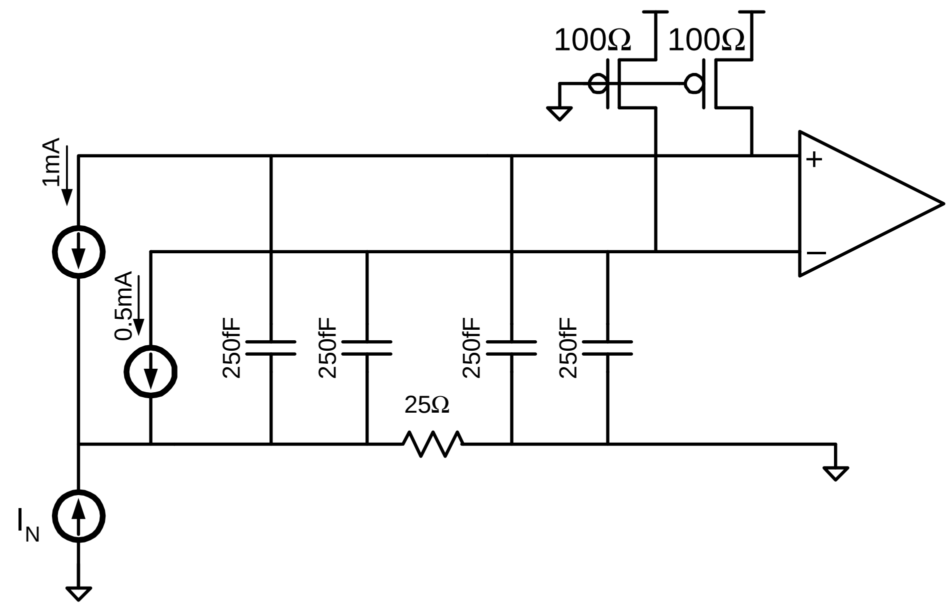


Here we see that the 10mA from I_N causes a 250mV drop across the 25W resistor. The 250mV 1ns ramp is input to a high-pass filter composed of two 250fF capacitors and a 100W resistor. The net *effect is the same as if 125mV were applied across a 500fF capacitor (which you get with a Tee network approximation). The resulting time constant is 500fF x 100W = 50ps. Since this is much faster than the 1ns ramp, the high-pass acts as a differentiator converting the 125mV/1ns ramp into a 6.25mV step as shown in the SPICE plot below. The net noise is V = 6.25mV.*



B. (6 points) What could you do to cancel the noise from part (A)?

The noise is probably small enough to ignore. However if you want to cancel it, the solution is to move the reference to the transmitter and run it over a matched line so it is contaminated by an identical 6.25mV of power supply noise. In this case the noise on the signal and the reference cancel..


100Ω
100Ω
1mA
0.5mA
250fF
250fF
250fF
250fF
25Ω
I_N
+
–