EE273 Digital Systems Engineering
Midterm Exam
February 13^{th}, 2002

SOLUTIONS

(Total time = 120 minutes, Total Points = 100)

Name: (please print)______________________________

In recognition of and in the spirit of the Stanford University Honor Code, I certify that I will neither give nor receive unpermitted aid on this exam.

Signature:_______________________________________

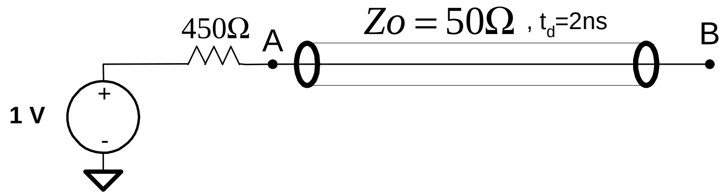
This examination is open notes open book. You may not, however collaborate in any manner on this exam. You have two hours to complete the exam. Please do all of your work on the exam itself. Attach any additional pages as necessary.

Before starting, please check to make sure that you have all 8 pages.

1		**40**
2		**20**
3		**25**
4		**15**
Total		**100**

Problem 1: Short Answer (40 Points: 10 questions, 4 points each)

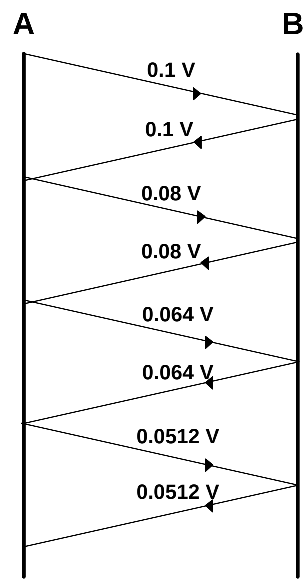
A. A voltage source with an ouput impedance of 450-Ohms applies a 1V step to a 50-Ohm transmission line that is 2ns long. The far end of the line is open. Immediately after the step what is the voltage across the source end of the line?



The voltage at point A immediately after the 1 V step is applied is:

$$1V \times \frac{50\Omega}{50\Omega + 450\Omega} = 0.1V$$

B. For the system of (A), how long after the step does the far end of the line reach 0.5V?



The termination is open, so we get complete reflection of the incident wave – i.e. the reflection coefficient at the termination is $k_{rt} = 1$. The reflection coefficient at the source is: $k_{rs} = \frac{450\Omega - 50\Omega}{450\Omega + 50\Omega} = 0.8$ *From the reflection diagram, we find that it takes 7 bounces before the voltage at point B reaches at 0.5 V. At (7x2ns) = 14ns, the voltage at point B reaches 0.59 V.*

C. Consider a transmission line with an impedance of 50-Ohms, a delay of 4ns, and an inductance of 300nH/m. Suppose a 10nH inductance is inserted in series every cm along this line. What are the delay and impedance of the line with inductors inserted? (Ignore for now the additional capacitance that would be associated with any real inductor.)

The capacitance of the line before adding the series inductance can be solved from the relationship:

$\sqrt{\frac{L}{C}} = 50\Omega$ *Plugging in for the known inductance per unit length, 300 nH/m, we get the capacitance per unit length: C = 120pF/m. We solve for the length of the line by solving for the velocity.*

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{300\frac{nH}{m} \times 120\frac{pF}{m}}} = 1.67 \times 10^8 \frac{m}{s}$$

Solving for the length of the line, we have: $length = v \times t_d = 1.67 \times 10^8 \frac{m}{s} \times 4ns = 0.67m$

Now we solve for the new delay and impedance of the line after inserting 10nH series inductors every cm. $L_{new} = L_{old}$ *+ 10nH/cm = 300nH/m + 10nH/cm = 1300nH/m.*

The new impedance is: $Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{1300\frac{nH}{m}}{120\frac{pF}{m}}} = 104\Omega$

The new velocity is: $\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1300\frac{nH}{m} \times 120\frac{pF}{m}}} = 8\times10^7\frac{m}{s}$

The new delay of the line is: $t_d = \frac{length}{v} = \frac{0.67m}{8\times10^7\frac{m}{s}} = 8.33ns$

D. For the transmission line of (C), what is the fastest rise time for which modeling this line as a uniform transmission line will give accurate results?

We use the equation: $t_r > 4t_s$*, where* t_s *is the delay of the each segment separated by the discontinuity.* $t_s = \frac{0.01m}{8\times10^7\frac{m}{s}} = 125ps$*. Thus, the fastest rise time is:*

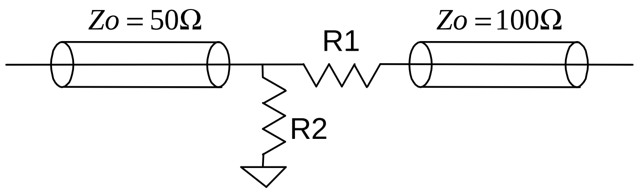
$t_r > 4(125ps) = 500ps$*. (Note: we also gave credit if you used the relationship: :* $t_r > 5t_s$ *.)*

E. Each conductor of a differential transmission line in a homogeneous medium has an odd-mode impedance of 50-Ohms and a total capacitance of 100pF/m. Of this 100pF/m, 20pF is capacitance to the other line of the differential pair. What is the even-mode impedance of the line?

Using the relationship: $Z_{odd} = \sqrt{\frac{L-M}{C+C_d}} = 50\Omega$ *we solve for the inductance and mutual inductance of the line. Plugging in the values for C (100pF/m) and* C_d *(20pF), we get: L-M = 300nH/m. Using the relationship* $\frac{L}{M} = \frac{C}{C_d}$ *we get: L = 5M. (Note: we can also obtain this relationship from the fact that the even-mode velocity and the odd-mode velocity are equal in a homogeneous medium.) Using these two equations to solve for L and M, we get: L=375nH/m, M=75nH/m. Now, we solve for the even-mode impedance of the line:*

$$Z_{even} = \sqrt{\frac{L+M}{C-Cd}} = \sqrt{\frac{450\frac{nH}{m}}{80\frac{pF}{m}}} = 75\Omega$$

F. Sketch a resistor network that can be placed between a 50-Ohm transmission line and a 100-Ohm transmission line so that waves propagating in either line toward the network will be passed on to the other line with no reflection. Make sure to show all component values.

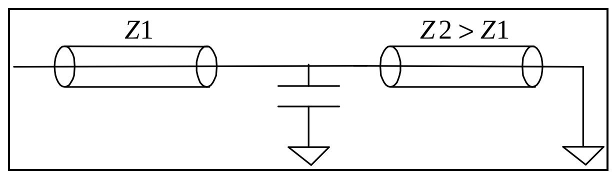


To pass a wave traveling in either direction without reflections, we solve the two equations:

$\boxed{100\Omega = R1 + 50\Omega // R2}$ $\boxed{50\Omega = R2 // (R1 + 100\Omega)}$

Solving these equations simultaneously, we get: $\boxed{R1 = R2 = 70.7\Omega}$

G. Sketch a qualitative circuit that will produce the following trace on a time-domain reflectometer (TDR). Just draw the type of components. Do not include numerical component values.



H. A 60Ω line (the victim line) is surrounded by two identical lines. All of these lines have a near end (reverse) crosstalk coefficient of k_{rx} = 0.1 and a far-end (forward) crosstalk coefficient of k_{fx} = 0. All three lines are terminated at both ends with 40Ω resistors. As a fraction of signal swing, how much noise will occur on the victim line (worst case) due to crosstalk? (you need give no more than two significant figures in your answer.)

At the near-end, each aggressor induces near-end cross-talk of 0.1. Thus, the total near-end cross talk induced on the victim by the two aggressor lines is k_{rx}*=0.2. At the near-end we have a reflection coefficient of* $\boxed{k_r = \frac{40-60}{40+60} = -0.2}$ *Thus, the near-end cross talk reflected to the far-end is:* $k_{rx}\, k_r$ *= (0.2)(-0.2) = -0.04. At the far-end this reflected wave will be reflected by the same reflection coefficient,* k_r*. Thus, the total near-end cross talk that reaches the far-end is -0.04(1+kr) = -0.04(1-0.2) = -0.032*

At the far-end, the 2 aggressor waves reflect off the mismatched termination resistor with a reflection coefficient, k_r *= -0.2. The combined effect of the reflected waves on each of the aggressors induces near-end cross talk at the far-end of the victim line with coefficient: :* $k_{rx}\ k_r$ *= (-0.2)(0.2) = -0.04. This near-end cross talk at the far-end of the victim line will be reflected off the mismatched termination of the victim with a coefficient of –0.2. Thus, the total coefficient for the reflected wave inducing near-end cross talk at the far-end of the victim line is: -0.04(1+kr) = -0.04(1-0.2) = -0.032*

Adding these coefficients we get a total cross talk coefficient of:

$$\boxed{k = \left| -0.032 + (-0.032) \right| = 0.064}$$

Note, the reflected wave of the near-end cross talk reflecting off the far-end will be reflected off the near-end and return to the far-end but this amount is so small compared to the cross talk value above that it's negligible. Similarly, the far-end induced near-end cross talk reflecting off the *source and returning to the far-end is also negligible.*

I. A signaling system has a BER of 10^{-15}, proportional noise of $K_N = 0.3$, signal swing of 100mV, and fixed noise of 10mV. If you double the signal swing to 200mV, what is the new BER?

$BER = e^{-\frac{VSNR^2}{2}}$ *Solving for VSNR, we get:* $VSNR = 8.31 = \frac{V_{NM}}{V_{rms}}$

The net margin in the original system is:

$$V_{NM} = V_{GM} - V_n = 50mV - (10mV + 0.3 \times 100mV) = 10mV$$

Solving for the rms Gaussian noise: Vrms = 1.2mV.
If we double the signal swing, our new net margin will be:

$$V_{NM} = V_{GM} - V_n = 100mV - (10mV + 0.3 \times 200mV) = 30mV$$

Thus, the new VSNR=24.9, and the new BER = 10^{-135}.

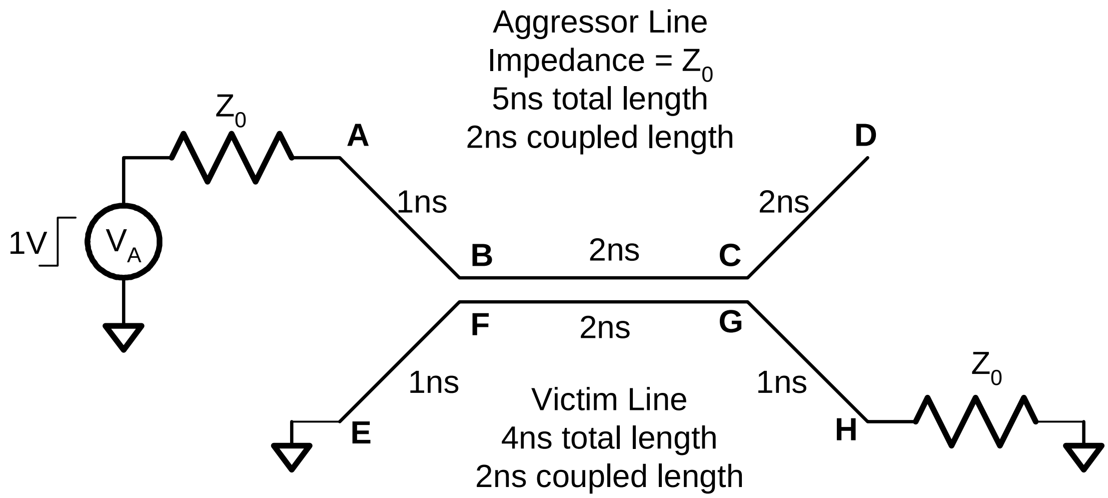
J. A chip has 4 single-ended signals sharing a 5nH return line. Each signal has a 1V swing and drives a 50-Ohm transmission line with a source impedance of 50-Ohms. What is the minimum rise time that will give total signal return cross talk of less than 0.1.

$$k_{XRT} = \frac{(N-1)Z_{RT}}{(N-1)Z_{RT} + Z_o + R_o} = \frac{3 \times \frac{5nH}{t_r}}{3 \times 5\frac{nH}{t_r} + 50\Omega + 50\Omega} < 0.1$$

Solving for t_r : $\boxed{t_r > 1.35ns}$

Problem 2: Transmission Lines (20 Points)

Consider the pair of coupled transmission lines shown below. The coupled section of the pair (segments BC and FG) has a near-end crosstalk coefficient k_{rx} of 0.1 and a far-end crosstalk coefficient, k_{fx} of 0. The aggressor line is driven directly by a 1V step source with a rise time of 100ps and a matched source impedance. The far end of the aggressor line is open. The victim line has its near end shorted to GND and its far end terminated into a matched impedance. (Note that the transmission line from C to D is twice as long as the line from G to H.)

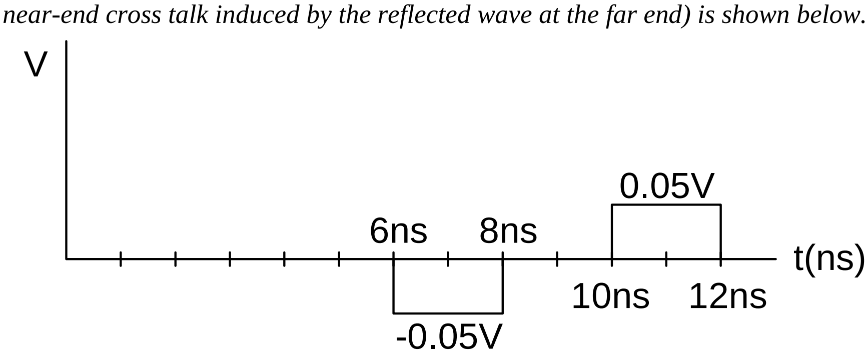


Using this information, sketch and dimension the voltage waveform at the far-end of the victim line (point H). You may ignore any effects that lead to waves with less than 10mV amplitude.

At time t=0, the 1V source on the aggressor line causes a 0.5V signal at point A. 1ns later, this 0.5V forward-traveling signal at point B induces 0.05 V near-end cross talk on the victim line at point F. 1ns later(at t=2ns) this reverse-traveling 0.05V near-end cross talk reaches point E. The short at point E reflects the near-end cross talk with a reflection coefficient of –1, giving a reflected wave of –0.05V. 4ns later (at t=6ns) the –0.05 near-end reflected wave reaches point H.

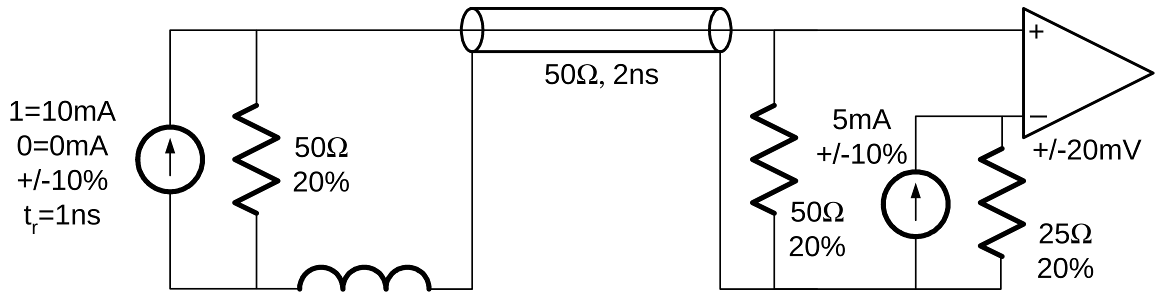
If the far-end of the aggressor were terminated with a matched resistance ($R_T = Z_o$) we would be done. However, the far-end of the aggressor is open and we get near-end cross talk at the far-end due to the reflected wave on the aggressor. At time t=5ns, the 0.5V incident wave reaches point D and is completely reflected (reflection coefficient = 1). Thus, we get a reverse-traveling wave of 0.5 V on the aggressor. 2ns later (at t=7ns), this reverse-traveling wave induces near-end cross talk at the far-end of the victim line of 0.05V. 1ns later (at t=8ns) this 0.05V reaches point H.

Since the lines are coupled for 2ns, each of the induced cross talk signals lasts for 4ns. The total voltage at point H, the superposition of these 2 induced waves (i.e. the reflected near-end cross talk seen at the far end and the near-end cross talk induced by the reflected wave at the far end) is shown below.



Problem 3: Signaling and Noise Analysis (25 points total)

Consider the unipolar, single-ended current-mode signaling system shown below. The transmitter represents a logic "0" as 0mA and a logic "1" as 10mA with ±10% error and has a rise time of t_r = 1ns. The line is terminated at both ends with resistors that have ±20% error. The reference is generated by a 5mA ± 10% current source and a 25-Ohm ±20% resistor. The receiver compares the reference and signal with ± 20mV offset. Also, each line is surrounded by two identical lines for purposes of crosstalk with a near-end crosstalk coefficient of k_{rx} = 0.1.



A. (5 points) What set of current source and resistor component values gives the worst possible performance (the greatest possible noise). List the values of each component. Make sure to include the values for the aggressor lines as well as for the victim line.

Victim current source	-10%	9mA
Current reference	+10%	5.5mA
Source resistance	-20%	40 Ω
Termination resistance	-20%	40 Ω
Reference resistance	+20%	30 Ω
2 neighboring (aggressor) current sources	+10%	11mA
2 neighboring (aggressor) source resistance	+20%	60 Ω
2 neighboring (aggressor) termination resistance	-20% (or +20%)	40 Ω

B. (15 points) List all of the bounded noise sources that affect this system and give the magnitude for each: in fraction of signal swing for proportional sources and in mV for fixed sources.

The nominal swing of our system is: 10mA x 25 Ω = 250mV

***Fixed noise**: 20mV receiver offset.*

***Reduced current drive** at the source: k = (10mA-9mA)/10mA = 0.1*

***Mismatched source termination:** voltage sent down the line is: 10mA x (40 Ω // 50 Ω) = 222mV. The swing should have been 10mA x 25 Ω = 250mV. Thus, the proportional noise is:*

$$k = \frac{250mV - 222mV}{250mV} = 0.112$$

Actually, the combined effect of the reduced current drive and mismatched source termination will not be 0.212, but it will be:

$$k = \frac{250mV - 9mA \times (40\Omega // 50\Omega)}{250mV} = 0.200$$

***Mismatched receiver termination:** At the far end the voltage is reduced by the reflection off the mismatched receiver termination. The 200mV incident wave is reduced by the k_r=-1/9 reflection to 178mV giving proportional noise of*

$$k = \frac{200mV - 178mV}{200mV} = 0.111$$

Of course the combined effects of the reduced current drive, the mismatched source and the mismatched receiver can all be considered together to give not .323 or .311, but

$$k = \frac{250mV - 178mV}{250mV} = 0.288$$

The -22mV reflected wave will reflect again off the source (k_r=-1/9) and return to the destination as a 2.4mV wave at the receiver. As this ISI due to mismatched terminations is less than 1% of the full swing signal we will ignore it.

***Reference current and termination offset:** The nominal voltage at the reference is: 5mA x 25 Ω =125mV. With the 10% error in current reference and the 20% error in the reference termination resistor, the voltage we get at the reference is now: 5.5mA x 30 Ω = 165mV. Thus the proportional noise caused by this noise source is:*

$$k = \frac{165mV - 125mV}{250mV} = 0.16$$

***Cross-talk:** The worst-case signal on each of the neighboring aggressors is: 11mA x (60Ω // 50Ω) = 300mV. This will induce a near-end cross talk on the victim line is: 2 x (300mV x 0.1) = 60mV. This near-end cross-talk will be reflected at the source of the victim line with a kr of (40-50)/(40+50)= -0.11. So, the amount of the near-end cross-talk reflected to the far end will be: (-0.111 x 60mV) = -6.67mV. This reflected wave will reach the far-end and again be reflected by –0.111. Thus, the magnitude of the near-end cross talk reaching the far-end will be: -6.67mV(1-0.111)=-5.93mV.*

At the far-end the forward-traveling waves on the aggressor lines are also reflected. The worst-case is that the reflected wave is: -0.111 x 300mV = -33.3mV. Thus, the near-end cross talk induced at the far *end of the victim line is: -33.3mV x 0.2 = -6.67mV. This cross-talk is reflected by the termination offset on the victim line. Thus the total magnitude of the near-end cross talk induced at the far-end is: -6.67mV(1-0.111) = -5.93mV.*

Adding, these two cross talk components together, we get a total cross talk noise at the far end of the victim line of –11.85mV. Thus, the proportional noise caused by this noise source is:

$$\boxed{k = \left|\frac{-11.85mV}{250mV}\right| = 0.047}$$

C. (5 points) Assuming 10mV of Gaussian noise, compute the net margin, VSNR, and BER for this signaling system.

The nominal signal swing is 250mV. Thus, we get the following values:

Adding up the proportional noise coefficents calculated in part B, we get:

$$k_N = 0.288 + 0.16 + 0.047 = 0.495$$

$$V_{GM} = \frac{250mV}{2} = 125mV$$

$$V_N = V_{ni} + k_N V_{swing} = 20mV + 0.495(250mV) = 144mV$$

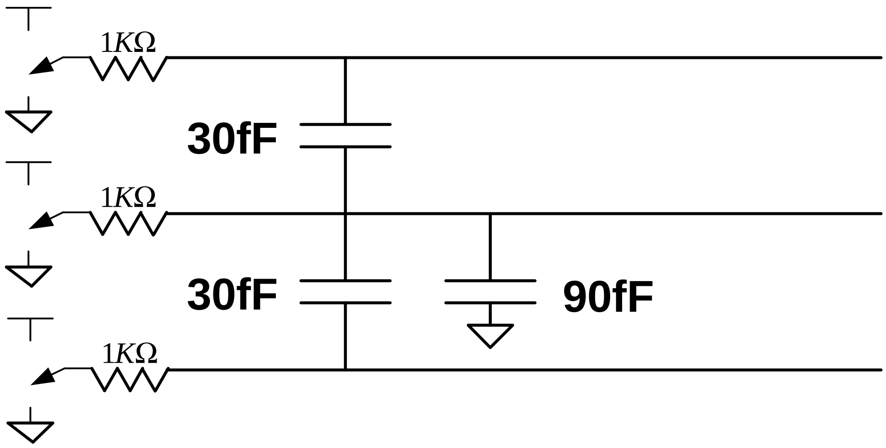
$$\boxed{V_{NM} = V_{GM} - V_N = 125mV - 144mV = -19mV}$$

VSNR = V_{GN}/V_{NM} is not defined with a negative net margin. Nor is BER.

Problem 4: Signaling over Lumped Loads (15 Points Total)

Consider an on-chip signaling system in which a voltage-mode driver with 1K-Ohm output impedance drives a 1mm wire. The wire has a total capacitance of 150fF of which 30fF is to the adjacent line to the left and 30fF is to the adjacent line to the right. The remaining 90fF of capacitance is to ground (and perpendicular signals that we approximate as ground).

A. (7 points) Consider three adjacent wires using this signaling system. What is the data sequence that will result in the middle line switching the fastest? (In this case, a data sequence is the state of the three lines before and after the transition. E.g., 000→011 is a data sequence – although not the one you want for this problem.) How fast does the middle line switch for this sequence?



The middle line will transition fastest if the two adjacent lines are switching in the same direction, effectively eliminating the 30fF coupling capacitances. Thus, the data sequences that will result in the middle line switching the fastest are: 111→000 *and* 000→111

This line will switch with a time constant of: $\tau = RCeff = 1K\Omega \times 90fF = 90ps$.

B. (8 points) What data sequence will result in the middle line switching the slowest? How fast does the middle line switch for this sequence?

The middle line will switch the slowest if the adjacent lines are transitioning in the opposite direction. When the adjacent lines are transitioning in the opposite direction from the middle line, the middle line effectively sees twice the coupling capacitance (60fF to each adjacent wire) due to the miller effect. Thus, the data sequences that will result in the middle line switching the slowest are: 101→010 *and* 010→101

The line will switch with a time constant of: : $\tau = RCeff = 1K\Omega \times 210fF = 210ps$